修士論文要旨

A mathematical model for the influence of the social insensitivity on the SIS epidemic dynamics

Ying XIE
東北大学大学院情報科学研究科 情報基礎科学専攻

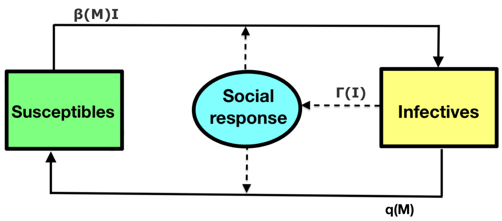
1 Introduction

The community sensitive to a disease spread generates the social response such as wearing a mask, limiting the number of contacts with others, taking medication, vaccination. Such behavioral changes may reduce the susceptibility to the disease or increase the recovery rate from it. However, the community may not respond to a transmissible disease even though such a disease is spreading in the community. A mathematical model is proposed and analyzed to consider the influence of the social insensitivity to the spread of a transmissible disease, while the infection rate and the recovery rate are affected by the social response to the disease spread.

2 Assumptions

- The disease is infatal and the disease-induced death can be negligible (for example, the common cold).
- The *social insensitivity* could be caused by the weak influence of the corresponding alert or by the unconcern to the disease spread, which are affected by the education, the culture, and the history of the community.
- The recovered individual can not get effectively long-lasting immunity and becomes susceptible again in a certain period after the recovery.
- The demographic change of the community is negligible in the time scale of considered epidemic dynamics.
- The effect of *social response* appears as the reduction of infection rate and the increase of recovery rate. For example, the social response may result in a decrease of individual contacts.
- The social response has a decay rate in time, while the existence of infectives in the community may enhance it.
- The disease spread may not enhance the social response if the number of infectives is small enough to make the people unconcern about it, that is, to cause the social insensitivity.

3 SIS modeling with social response



Susceptibles (S): individuals who are healthy and can be infected.

Infectives (I): individuals who have been infected and are able to transmit the infection.

M: the strength of the social response.

We can derive

$$\begin{aligned}\frac{dS}{dt} &= -\beta(M)IS + q(M)I\\ \frac{dI}{dt} &= \beta(M)IS - q(M)I\\ \frac{dM}{dt} &= \Gamma(I) - \mu M.\end{aligned}$$

$\beta(M)$: the infection coefficient which is a continuous and decreasing function of M with $\beta(0) = \beta_0 > 0$;

$q(M)$: the recovery rate which is a continuous and increasing function of M with $q(0) = q_0 > 0$;

$\Gamma(I)$: the social sensitivity function with $\Gamma(I) \geq 0$;

$$\Gamma(I) := \begin{cases} 0 & \text{for} \quad I \leq I_c; \\ \gamma(I - I_c) & \text{for} \quad I > I_c. \end{cases}$$

γ: the social sensitivity;

I_c: the threshold value for the number of infectives to cause the social response.

μ: the decay rate of the social response;

N: the total population size in the community which is given as $S(t) + I(t) = N > 0$ for any $t \geq 0$.

$\theta_c := I_c/N$. The basic reproduction number (基本再生産数) $\mathscr{R}_0$: the expected number of secondary infectives who is infected, in a totally susceptible community, by a single infected individual during the time span of the infection. For the generic model,

$$\mathscr{R}_0 := \frac{\beta_0 N}{q_0}.$$

4 Analytical result

Theorem

(i) *If and only if* $\mathscr{R}_0 \le 1$, *there is the unique disease-free equilibrium* $E_0(0,0)$, *which is globally asymptotically stable.*

(ii) *If and only if* $1 < \mathscr{R}_0 \le (1-\theta_c)^{-1}$, *there are two equilibria: the disease-free equilibrium* $E_0(0,0)$ *and the endemic equilibrium* $E_{+0}\big(N(1-\mathscr{R}_0{}^{-1}),0\big)$, *of which* E_0 *is unstable, while* E_{+0} *is globally asymptotically stable.*

(iii) *If and only if* $\mathscr{R}_0 > (1-\theta_c)^{-1}$, *there are two equilibria: the disease-free equilibrium* $E_0(0,0)$ *and the endemic equilibrium* $E_{++}(I^*,M^*)$, *of which* E_0 *is unstable, while* E_{++} *is globally asymptotically stable.*

Corollary *When* $\mathscr{R}_0 \le (1-\theta_c)^{-1}$, *the system monotonically approaches the equilibrium.*

5 A specific model

We give specific functions for $\beta(M)$ and $q(M)$:

$$\beta(M) = \frac{\beta_0}{1+aM}; \quad q(M) = q_0 + bM.$$

We define non-dimensional transformation of variables and parameters given by

$$u = \frac{S}{N}; \quad v = \frac{I}{N}; \quad \tau = q_0 t; \quad \eta = \frac{N\gamma}{q_0};$$

$$B = \frac{b}{q_0}; \quad \delta = \frac{\mu}{q_0}; \quad \mathscr{R}_0 = \frac{\beta_0 N}{q_0}.$$

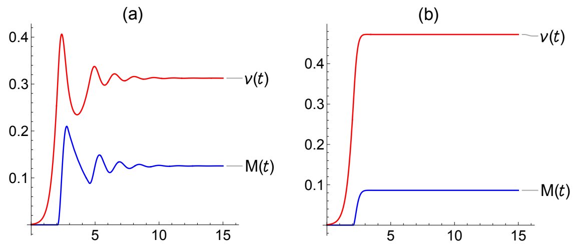
The system can be rewritten as follows:

$$\begin{aligned}\frac{du}{d\tau} &= -\frac{\mathscr{R}_0}{1+aM}uv + (1+BM)v\\ \frac{dv}{d\tau} &= \frac{\mathscr{R}_0}{1+aM}uv - (1+BM)v\\ \frac{dM}{d\tau} &= G(v) - \delta M,\end{aligned}$$

where

$$G(v) := \begin{cases} 0 & \text{for} \quad v \le \theta_c; \\ \eta(v-\theta_c) & \text{for} \quad v > \theta_c, \end{cases}$$

and $u(t)+v(t) \equiv 1$ for any $t \ge 0$. The behavior to approach the endemic equilibrium has two different manners: damped-oscillatory or monotonic.



Numerical examples of the temporal variation with $(v(0), M(0)) = (0.001, 0)$; $\theta_c = 0.3$; $\mathscr{R}_0 = 4.0$; $a = 5.0$; $B = 5.5$; $\eta = 5.0$; (a) $\delta = 0.5$; (b) $\delta = 10.0$.

6 Conclusion

- When there is no social response, the system becomes the standard and simplest SIS model, and then shows a monotonic approach to an endemic equilibrium.
- If the community is more insensitive to the disease, the endemic size becomes larger.
- The larger decay rate of the social response increases the endemic size, and that the more sensitive social response makes the endemic size smaller.
- The more sensitive social response is more likely to cause a damped oscillation.
- The social response may play an important role to cause repetitive outbreaks in epidemic dynamics.

Doctor Thesis

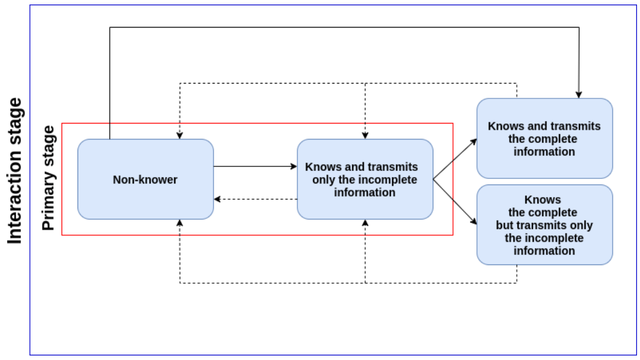
Population dynamics modeling for the effect of collective behavior on information spread

Emmanuel Jesuyon DANSU

Tohoku University, Sendai, Japan

We consider two major models, namely a rejoinder model in which there are two interacting pieces of information spreading with a time lag between them and a threshold model in which a person only begins to spread a piece of information after an acceptable number of people have been spreading it.

In the rejoinder model, we employ the use of a system of ordinary differential equations to see how two competing pieces of information behave at a given time, t, in a population divided into those who have not been exposed to any of the two pieces of information, $U(t)$; those who know and transmit only the first piece of information, $P_1(t)$; those who know and transmit only the second piece of information, $P_2(t)$; those who know both but transmit only the first piece of information, $V_1(t)$; those who know both but transmit only the second piece of information, $V_2(t)$.



Going by these five groups, we have the following general formulas for our model:

$$\frac{dU}{dt} = -\Lambda_1 U - \Lambda_2 U - \Lambda_3 U - \Lambda_4 U - \Lambda_5 U;$$

$$\frac{dP_1}{dt} = \Lambda_1 U - \Gamma_1 P_1 - \Gamma_2 P_1 - \Gamma_3 P_1;$$

$$\frac{dP_2}{dt} = \Lambda_2 U - \Xi_1 P_2 - \Xi_2 P_2 - \Xi_3 P_2;$$

$$\frac{dV_1}{dt} = \Lambda_3 U + \Gamma_1 P_1 + \Xi_1 P_2 - \Psi_1 V_1 + \Psi_2 V_2 - \Phi_1 V_1;$$

$$\frac{dV_2}{dt} = \Lambda_4 U + \Gamma_2 P_1 + \Xi_2 P_2 + \Psi_1 V_1 - \Psi_2 V_2 - \Phi_2 V_2.$$

Furthermore, we have those who transfer directly from the state U to a state in which they know and transmit both pieces of information, $W_{02}(t)$; those who transfer from the states P_1 and P_2 to a state where they know and transmit both pieces of information, $W_{12}(t)$; those who transfer from the states V_1 and V_2 to a state where they transmit both pieces of information, $W_{22}(t)$ such that we have

$$\frac{dW_{02}}{dt} = \Lambda_5 U;$$

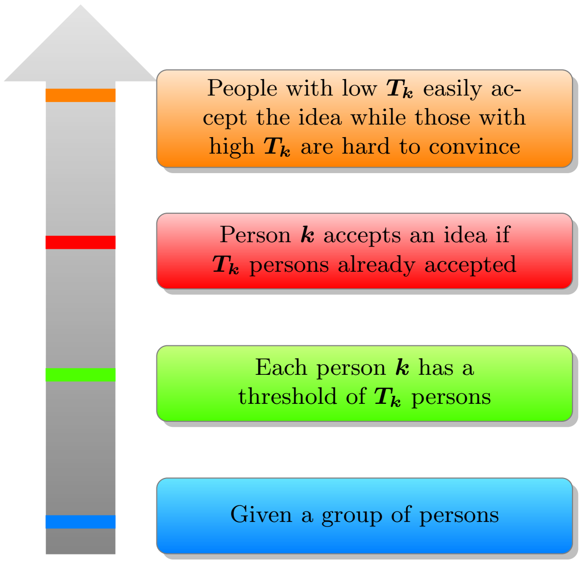
$$\frac{dW_{12}}{dt} = \Gamma_3 P_1 + \Xi_3 P_2;$$

$$\frac{dW_{22}}{dt} = \Phi_1 V_1 + \Phi_2 V_2.$$

The coefficients Λ_i, Γ_i, Ξ_i, Ψ_i and Φ_i are related to transition of states and they are generally functions of relevant subpopulations. We consider this eight dimensional system in order to understand the dynamics of various population segments with respect to the spread of two items of information with unique attributes.

The threshold model is based on the idea of Mark Granovetter who promoted the threshold model of social behavior in which the acceptance value of one of two distinct actions is determined by the proportion of a given population that has already accepted the action. We consider the possibility of an individual accepting and spreading some information given that a satisfactory proportion of people (threshold population) in their community is already doing the same. Given the frequency/proportion $P(t)$ of knowers of the information at a given time and the strength of social recognition effect $Q = Q(P)$ of the information, we assume that each individual is characterized by a threshold value ξ for Q, independent of time, such that

$$\begin{cases} \xi \leq Q \rightarrow \text{The individual may accept the information to transmit to others;} \\ \xi > Q \rightarrow \text{The individual ignores the information.} \end{cases}$$



The differential equation model representing this information spread behavior is given as

$$\frac{dP}{dt} = B(P(t)) \left[1 - P(t) - \int_{\Xi(P(t))} \{1 - \theta(\xi)\} f(\xi) d\xi \right],$$

where $B(P)$ is the coefficient of information transmission to non-knowers who are willing to accept the information; $\Xi(P)$ is the set of threshold values for which people are not yet willing to accept the information; $\theta(\xi)$ determines the ratio of initial knowers in the subpopulation with the threshold value ξ, such that $0 \leq \theta(\xi) \leq 1$; $f(\xi)$ is the frequency distribution function (FDF) for the threshold value ξ in the population. Our analyses show that the final proportion of knowers of the information is determined by the initial proportion of knowers. We also see the existence of critical values for the initial knower size, the mean threshold value and the variance of threshold values. These critical values tend to have drastic impact on the proportion of the population that end up knowing the information.

The findings from this thesis could help us to appreciate the impact of misinformation on the society and promote information literacy which has become a very crucial need in this internet-enabled information age.

References

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